Divisibility

Definition. **Multiples** and **divisors (factors)**

A natural number '*a*' is multiple of another natural number '*b*' when '*a*' contains '*b*' an **exact** number of times.

It is also said that '*b*' is divisor (or factor) of '*a*'. It is also said that '*a*' is **divisible** by '*b*'.

For example 24 is multiple of 6, because 24 contains 6 four times, we can also say: 6 is divisor (or factor) of 24 or that 24 is divisible by 6.

We can get the multiples of a number multiplying this number by natural numbers, for example, we obtain the multiples of 6:

$$6 \cdot 1 = 6,\ 6 \cdot 2 = 12,\ 6 \cdot 3 = 18,\ \dots$$

To obtain the divisors of a number, we divide this number by smaller natural numbers and see if each division is exact or not. To get divisors of 30, we divide this number by 1, 2, 3, 4, …and we get exact divisions for the following numbers: 1, 2, 3, 5, 6, 10, 15 and 30, these numbers are the divisors (factors) of 30.

Prime and **composite** numbers

A number is prime if it is only divisible by 1 and itself, if a natural number is not prime is called composite. For example 13 is prime and 10 is composite.

Eratosthenes **sieve**

That is a way to **search** for prime numbers by **eliminating** multiples of the first prime numbers.

Divisibility **tests**

With these tests it is not necessary to make divisions to **check** divisibility. Remind 2, 3, 5 tests:

- A number is divisible by 2 when its last **digit** is 0 or an **even** digit.
- A number is divisible by 3 if we add its digits and the result is 3 or multiple of 3.
- A natural number is multiple of 5 when its last digit is 5 or 0.

Algorithm to check if a natural number is prime or not

To know if a number '*a*' is prime we divide this number by smaller prime numbers, if a division is exact the number is composite, if we don't **find** any exact division we make divisions by smaller prime numbers until the quotient is smaller than the divisor, when this happens we know that number '*a*' is prime.

Factorization

To **Factorize** a natural number is to express that number as a multiplication of powers of prime numbers. For example, the factorization of 72 is:

$$72 = 2^3 \cdot 3^2$$

Greatest common divisor and least common multiple (g.c.d. and l.c.m.)

They are also called **highest common factor** and **lowest common multiple**.
They are used in many Maths tasks, for example to **simplify** fractions and to obtain the **common denominator** of several fractions.
The way to obtain the g.c.d. and the l.c.m. of two or more natural numbers is: first obtain their factorization and then

· g.c.d. is the product of **common factors** with **lowest exponent**
· l.c.m. is the product of **common and not common prime factors** with their **highest exponent.**

Euclid Algorithm

This is a way to **achieve** the greatest common divisor of two numbers: the **bigger one** is divided by the **smaller one**. After this, we divide each quotient by each remainder and, when the remainder is 0, the last quotient is the greatest common divisor.
We can also obtain the least common multiple, applying the following formula:

$$l.cm. = \frac{a \cdot b}{g.c.d.}$$